Gyrokinetic Simulation of Turbulence and Transport with Kinetic Electrons and Finite β Effects

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ABSTRACT

Turbulence and transport in tokamaks are studied using a 3-dimensional gyrokinetic particle code that uses the generalized split-weight scheme for the electrons. The inaccuracy problem at high plasma β is solved by using the same marker particle distribution as is used for δf to evaluate the $\beta m_i/m_e A_\parallel$ term in Ampere's equation, which is solved iteratively. It is found that for H-mode parameters, the nonadiabatic effects of kinetic electrons increase linear growth rates of the Ion-Temperature-Gradien-Driven (ITG) modes, mainly due to trapped-electron drive. The ion heat transport is also increased from that obtained with adiabatic electrons. The linear behavior of the zonal flow is not significantly affected by kinetic electrons. The ion heat transport decreases to below the adiabatic electron level when finite plasma β is included due to finite-β stabilization of the ITG modes. This work is being carried out using the "Summit Framework."

Split-weight Scheme

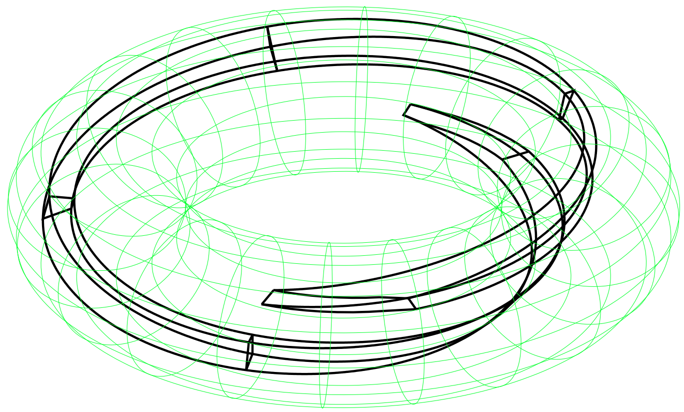
- Perturbation $\mathbf{E}_\perp = \nabla_\perp \phi$, $E_\parallel = \nabla_\parallel \phi - \dfrac{\partial A_\parallel}{\partial t}$, $\delta \mathbf{B}_\perp = \nabla A_\parallel \times \mathbf{b}$
- Use $p_\parallel = v_\parallel + \dfrac{q}{m} A_\parallel$ as a coordinate to eliminate $\dfrac{\partial A_\parallel}{\partial t}$. (Hahm'88)
- $f_e = f_{\mathrm{Me}} + \epsilon_g e \phi \dfrac{f_M}{T} + h$.
- Quasi-neutrality: $\tilde{\phi} = \Sigma_{\mathbf{k}} \Gamma_0(b) \phi_k \, \exp(i\mathbf{k}\cdot\mathbf{x})$

$$n_{0\mathrm{i}} \frac{q_i^2}{T_i}(\phi - \tilde{\phi}) + \epsilon_g n_{0\mathrm{e}} \frac{e^2}{T_e} \phi = \delta \bar{n}_i - \delta n_{\mathrm{eh}}$$

- Equation for $\partial \phi / \partial t$

$$n_{0\mathrm{i}} \frac{q_i^2}{T_i} \left(\frac{\partial \phi}{\partial t} - \frac{\partial \tilde{\phi}}{\partial t} \right) = -\nabla \cdot \int f_i \mathbf{v}_{\mathrm{Gi}} d\mathbf{v} + \nabla \cdot \int f_e \mathbf{v}_{\mathrm{Ge}} \, d\mathbf{v}$$

- Ampere's law $\left(-\nabla_\perp^2 + \frac{\omega_{\mathrm{pe}}^2}{c^2} \right) A_\parallel = \mu_0 \left(u_{\parallel\mathrm{i}} - u_{\parallel\mathrm{e}} \right)$



- $x = r - r_0,\ \ y = \frac{r_0}{q_0}(q\theta - \zeta), \quad z = q_0 R_0 \theta.$
- $l_z = 2\pi q_0 R_0$. Periodic in x-y, toroidal bc's.
- Predictor-Corrector (no electron sub-cycling and orbit averaging)
- Fourier method for gyrokinetic quasi-neutrality condition and Ampere's law
- 1D domain decomposition in z with domain cloning at each z. (C. Kim and S. E. Parker, 2000)

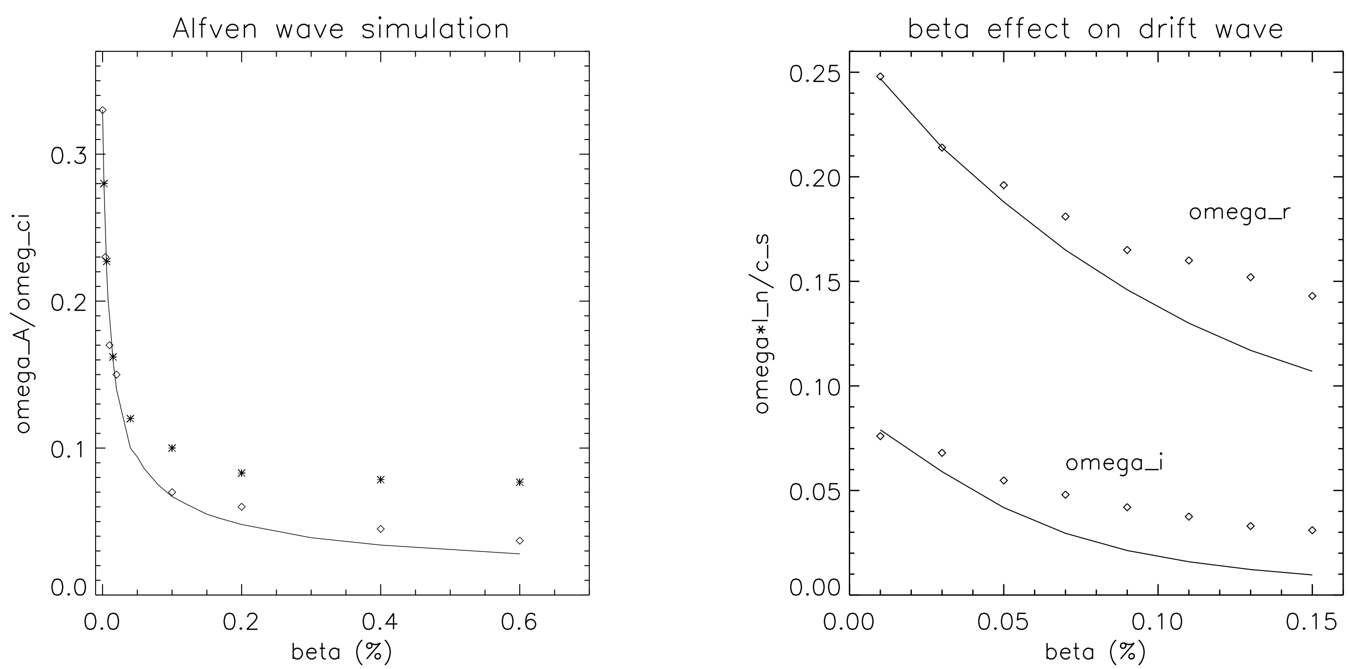
High β Problem and Solution

- For $k_\perp \rho_i \leq 1$ the $\frac{\omega_{\rm pe}^2}{c^2} = \beta_e \frac{m_i}{m_e}$ term much larger than the $k_\perp^2$ term in the Ampere's law.
- This term should be exactly cancelled linearly by a corresponding part of $u_{\|\rm e}$. However, this cancellation is difficult to attain, since $u_{\|\rm e}$ comes from particles and contains the effects of finite particle number and finite particle size

$$u_{\|\rm e}(\mathbf{x}) = \sum_j w_{\rm ej}\ v_{\|\rm j}\ S(\mathbf{x}_j - \mathbf{x})$$

  w_e electron weight. S is the particle shape. $\mathbf{x}$ location of the grid point, $\mathbf{x}_j$ the particle position.

- The problem is not caused by the split-weight scheme. Using $v_\|$ formulation with the same split-weight scheme does not solve the problem. However, $v_\|$ formulation with a different split-weight scheme (W. W. Lee, 2001) was demonstrated in low-dimensionality simulations to be free from this difficulty. It is not clear whether this is true in 3-D simulations.
- $v_\|$ formulation with finite differencing $\frac{\partial A_\|}{\partial t}$, even time centered, leads to numerical instability.


Alfven wave simulation
omega_A/omeg_ci
beta (%)
beta effect on drift wave
omega*l_n/c_s
omega_r
omega_i
beta (%)

Solution of the β Problem

- The $\beta_i \frac{m_i}{m_e}$ term in Ampere's law comes from $f_{0\mathrm{e}}(p_{\|})$, Maxwellian distribution in terms of $p_{\|}$

$$\delta j[f_{0\mathrm{e}}] = \frac{1}{m_e} \int f_{0\mathrm{e}}(v_\perp, p_{\|}) A_{\|}(\mathbf{x})\, d\mathbf{v} = \frac{1}{m_e} A_{\|}.$$

- The difference between $f_{0\mathrm{e}}(v_{\|})$ and $f_{0\mathrm{e}}(p_{\|})$ is represented in particle weights. The linear part of $f_{0\mathrm{e}}(v_{\|}) - f_{0\mathrm{e}}(p_{\|})$ is

$$f_{0\mathrm{e}}(v_{\|}) - f_{0\mathrm{e}}(p_{\|}) \approx -\tau p_{\|} A_{\|} f_{0\mathrm{e}}(p_{\|}),$$

  The current from this linear part is the same as that coming from $f_{\mathrm{oe}}(p_{\|})$ but in the opposite direction,

$$\int \left(-\tau p_{\|} A_{\|} f_{0\mathrm{e}}(p_{\|})\right)\ v_{\|}\ d\mathbf{v} = -\frac{1}{m_e} A_{\|}.$$

- Rewrite $\delta j[f_{0\mathrm{e}}]$ so that it has the same velocity dependence in the integral

$$\delta j[f_{0\mathrm{e}}] = \tau \int f_{0\mathrm{e}}(v_\perp, p_{\|})\, p_{\|}^2\, A_{\|}(\mathbf{x})\, d\mathbf{v}.$$

- Replace $f_{0\mathrm{e}}$ with its discrete representation (with proper normalization),

$$\widetilde{f}_{0\mathrm{e}} \approx \frac{V}{N} \frac{1}{2\pi v_\perp} \sum_j \delta(\mathbf{x} - \mathbf{x}_j)\, \delta(v_\perp - v_{\perp\mathrm{j}})\, \delta(v_{\|} - v_{\|\mathrm{j}}),$$

- The same scattering operation as that used for $\int h v_\parallel \, d\mathbf{v}$ has to be used to distribute $A_\parallel$ at the particle location to nearby grid points,

$$\widetilde{\delta j}[f_{0\mathrm{e}}](\mathbf{x}) \approx \frac{V}{N} \tau \sum_j p_{\parallel \mathrm{j}}^2 \, A_\parallel(\mathbf{x}_j) \, S(\mathbf{x} - \mathbf{x}_j).$$

- The value of $A_\parallel$ at the particle location $\mathbf{x}_j$ is calculated from the values at the neighboring grids using the same shape function,

$$A_\parallel(\mathbf{x}_j) = \sum_{\mathrm{l,m,n}} A_\parallel(\mathbf{x}_{\mathrm{l,m,n}}) S(\mathbf{x}_j - \mathbf{x}_{\mathrm{l,m,n}}),$$

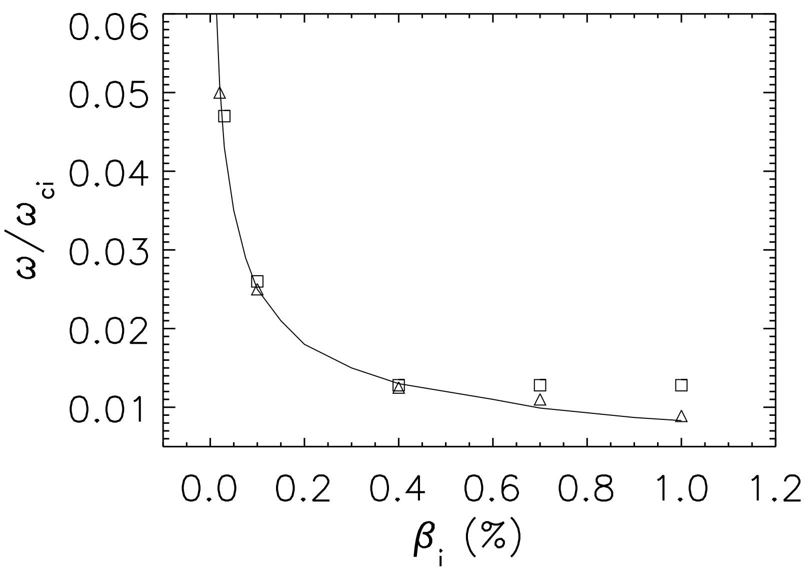
- Rewrite Ampere's law as

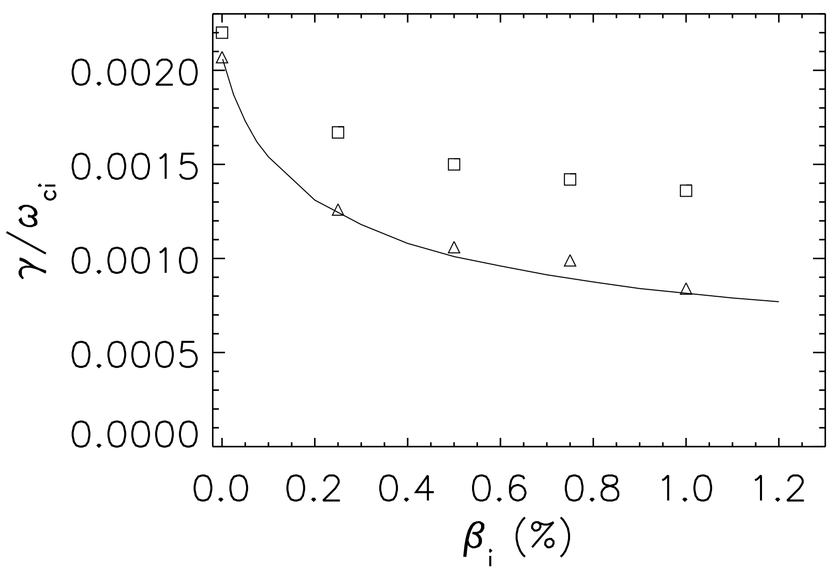
$$\left(-\nabla_\perp^2 + \frac{\beta_i}{m_e}\right) A_\parallel^{\mathrm{n+1}} = \beta_i(\widetilde{\delta u}_{\parallel \mathrm{i}} - \widetilde{\delta u}_{\parallel \mathrm{e}}) + \beta_i \left(\frac{1}{m_e} A_\parallel^n - \widetilde{\delta j}[f_{0\mathrm{e}}]\right),$$

- Number of iteration $5 \sim 7$.
- It is found that similar technique for the $\epsilon_g n_{0\mathrm{e}} \frac{e^2}{T_e} \phi$ term in the quasi-neutrality condition is not necessary, since $\epsilon_g \leq 1$.

Finite β Effects on Slab ITG - Good Agreement Between Simulation and Dispersion Relation

Slab dispersion relation $-k_\perp^2 \frac{k_\parallel}{\omega}(M_i - M_e - {k'}_\perp^2) = \beta(N_e - N_i)({k'}_\perp^2 - L_i + L_e)$ with $L_i = (\Omega - \Omega_{\rm Ti})\Gamma_0 + \Omega_{\rm Ti}\Gamma_*$, $M_i = -\Gamma_0(1+\zeta_i Z_i) + (\frac{3}{2}\Omega_{\rm Ti}\Gamma_0 - \Omega_{\rm Ti}\Gamma_* - \Omega\Gamma_0)\zeta_i Z_i - \Omega_{\rm Ti}\Gamma_0\zeta_i^2(1+\zeta_i Z_i)$, $N_i = -\frac{\omega}{k_\parallel}[\Omega\Gamma_0(1+\zeta_i Z_i) + (-\frac{3}{2}\Omega_{\rm Ti}\Gamma_0 + \Gamma_0 + \Omega_{\rm Ti}\Gamma_*)(1+\zeta_i Z_i) + \Omega_{\rm Ti}\Gamma_0(\frac{1}{2} + \zeta_i^2 + \zeta_i^3 Z_i)]$, $L_e = \Omega$, $M_e = 1 - (\Omega - \Omega_{\rm Te}/2 - 1)\zeta_e Z_e(\zeta_e) - \Omega_{\rm Te}\zeta_e^2(1+\zeta_e Z_e)$, $N_e = -\frac{\omega}{k_\parallel}[\Omega_{\rm Te}(\frac{1}{2} + \zeta_e^2(1+\zeta_e Z_e)) + (\Omega - \Omega_{\rm Te}/2 - 1)(1+\zeta_e Z_e)]$, $\kappa_n = -\frac{dn_0}{dx}/n_0$, $\kappa_{\rm T\alpha} = -\frac{dT_{0\alpha}}{dx}/T_0$, $\Omega = \kappa_n k_y/\omega$, $\Omega_{\rm T\alpha} = \kappa_{\rm T\alpha} k_y/\omega$, $\Gamma_0 = \Gamma_0(b) = \Gamma_0(k_\perp^2 v_{\rm ti}^2/\Omega_i^2)$, $\Gamma_* = \Gamma_0 - b(\Gamma_0 - \Gamma_1)$, ${k'}_\perp^2 = 1 - \Gamma_o(b)$. $\zeta_\alpha = \omega/\sqrt{2}k_\parallel v_{\rm t\alpha}$. $Z_\alpha = Z(\zeta_\alpha)$ the plasma dispersion function.





$k_x\rho_i = 0.2$, $k_y\rho_i = 0.4$, $k_\parallel\rho_i = 7.14\times10^{-4}$, $\kappa_T\rho_i = 0.2$, $\kappa_n\rho_i = 0.04$, $\kappa_{\rm Te}\rho_i = 0$, $32\times32\times32$, 262 144 e's and i's, $\epsilon_g = 0.5$,

$l_x \times l_y \times l_z = 32\rho_i \times 32\rho_i \times 8796\rho_i$, $\triangle t\,\omega_{\rm ci} = 2$

Electron-ion Collisions

- Lorentzian operator

$$C_L(f_e) = \nu_e \frac{1}{2}\frac{\partial}{\partial\lambda}(1-\lambda^2)\frac{\partial}{\partial\lambda} f_e$$

with

$$\nu_e = \frac{n_{0\mathrm{e}} e^4 \ln\Lambda}{4\pi\epsilon_0^2 m_e^2 v^3}\left(Z_{\mathrm{eff}} + H_{\mathrm{ee}}\left(\sqrt{m_e v^2/2T_{0\mathrm{e}}}\right)\right),$$

$$H_{\mathrm{ee}}(x) = \frac{e^{-x^2}}{\sqrt{\pi}x} + (1 - \frac{1}{2x^2})\mathrm{erf}(x).$$

- Expand $C_L(f_e)$ as

$$C_L(f_e) = C_L(f_{0\mathrm{e}}(p_\|)) - C_L(\epsilon_g \phi \frac{\partial f_{0\mathrm{e}}}{\partial \varepsilon_e}) + C_L(h).$$

The ϵ_g term is nonlinear and will be neglected. The first term is given by,

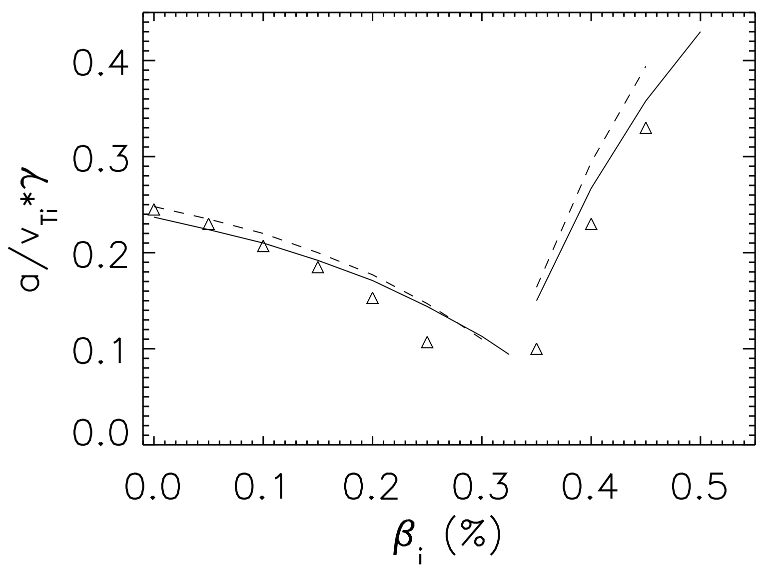
$$C_L(f_{0\mathrm{e}}(p_\|)) = -\tau \nu_e A_\| f_{0\mathrm{e}},$$

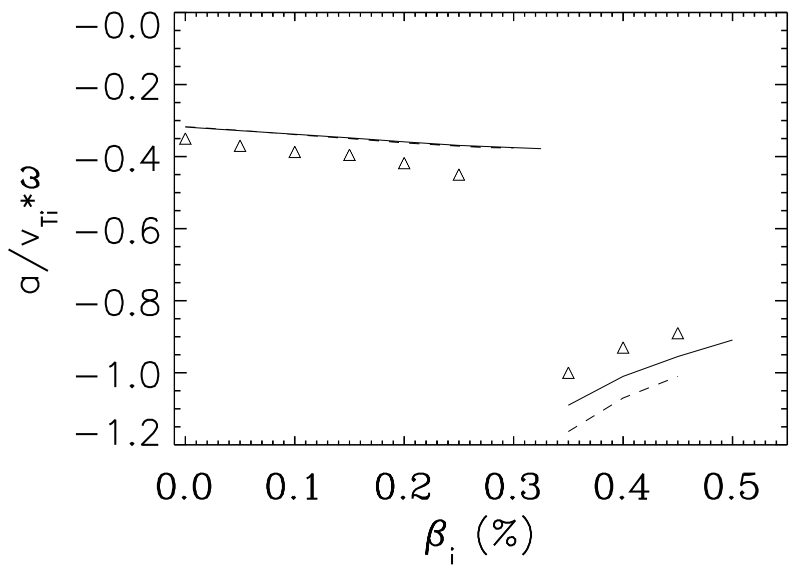
implemented as an additional term in the electron weight equation. $C_L(h)$ is implemented using the Monte-Carlo method

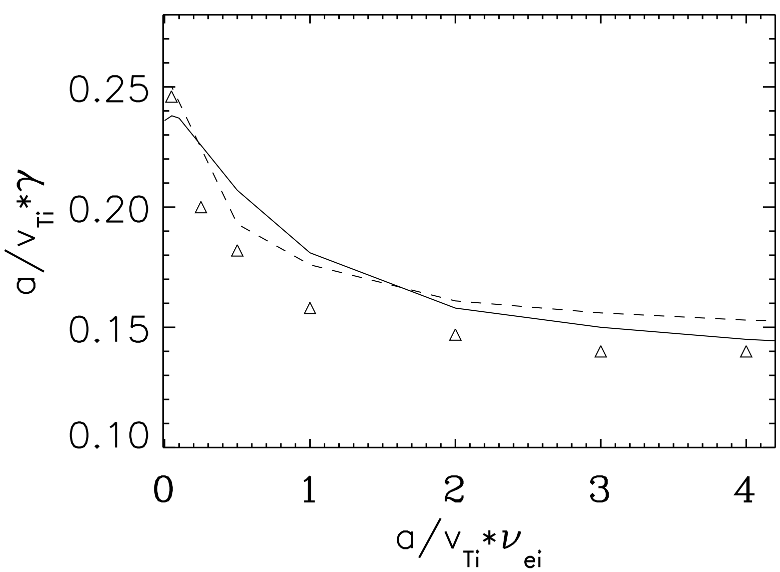
$$\lambda_{\mathrm{new}} = \lambda_{\mathrm{old}}(1-\nu_e\delta t) \pm \left[\left(1-\lambda_{\mathrm{old}}^2\right)\nu_e\delta t\right]^{1/2},$$

where $\pm$ means equal probability of $+$ or $-$ (Boozer and Kuo-Petravic, '81). $\delta t = \triangle t$ for corrector step and $\delta t = 2\triangle t$ for predictor step, $\triangle t$ is the time step.

Linear Benchmarking with gks and GYRO

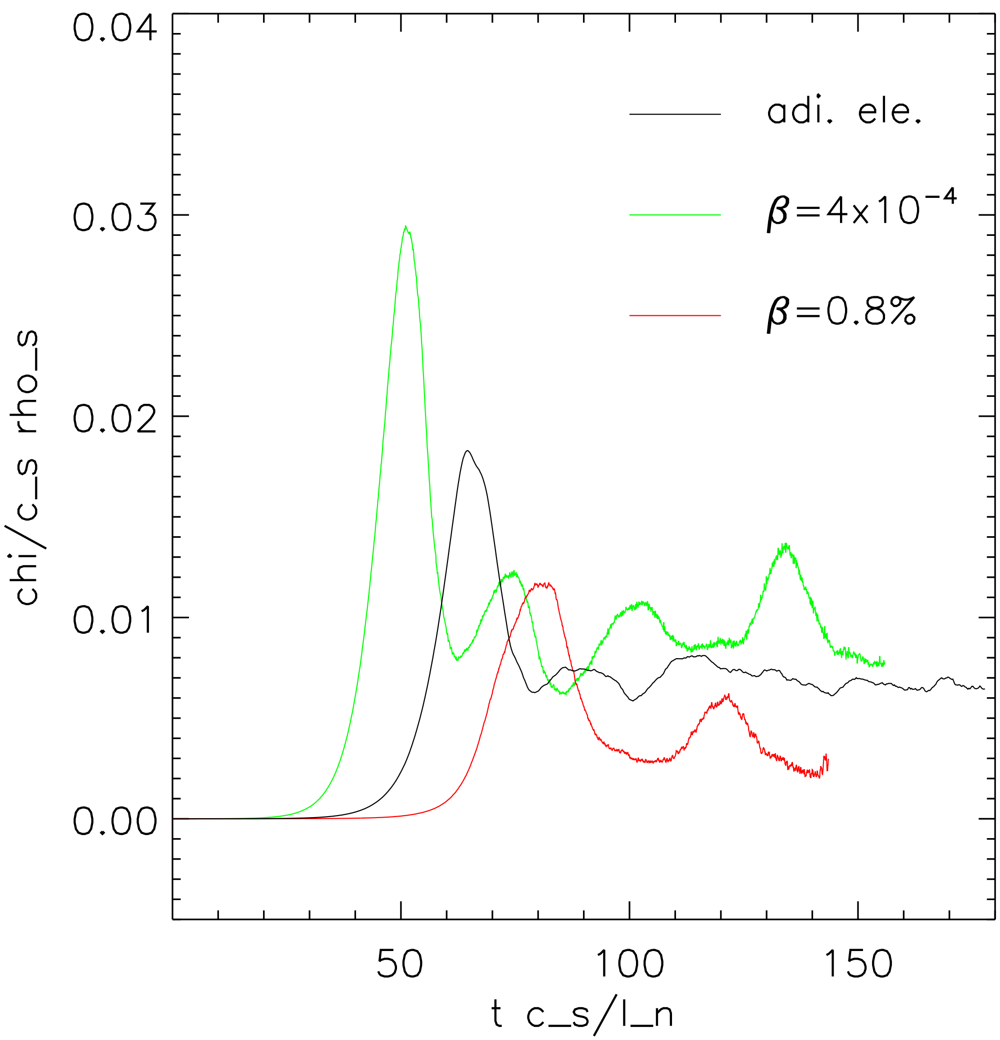






Compare particle code with continuum code for the Waltz Standard Case. (a) Mode growth rate vs. β_i; (b) mode frequency vs. β_i and (c) mode growth rate vs. collision rate

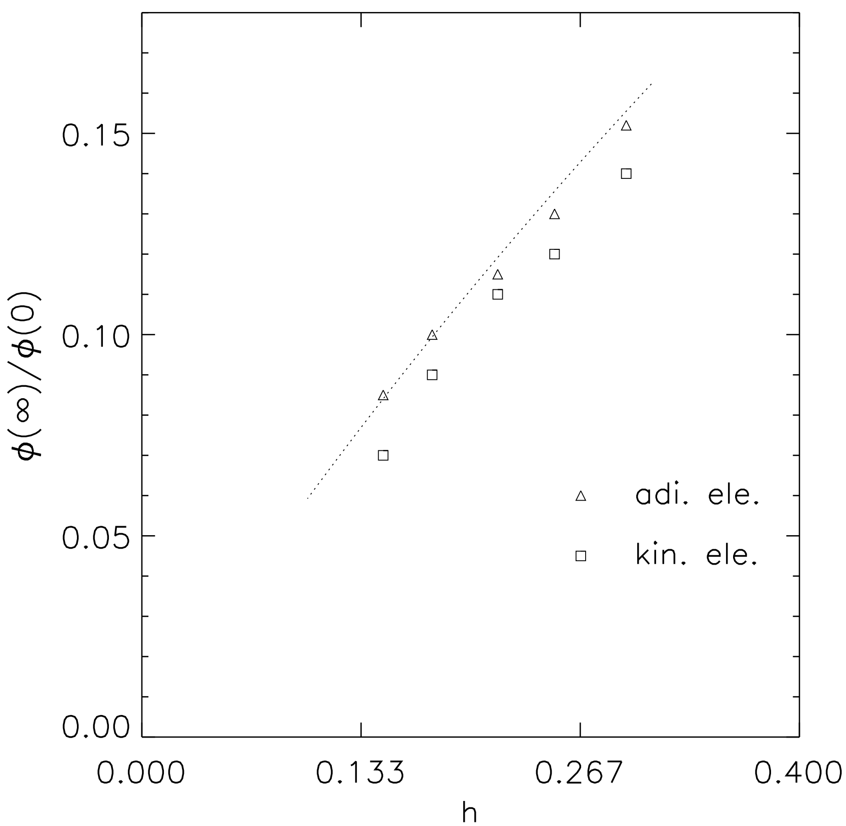
Ion Heat Flux Decreases below Adiabatic Level due to β Stabilization of the ITG Modes

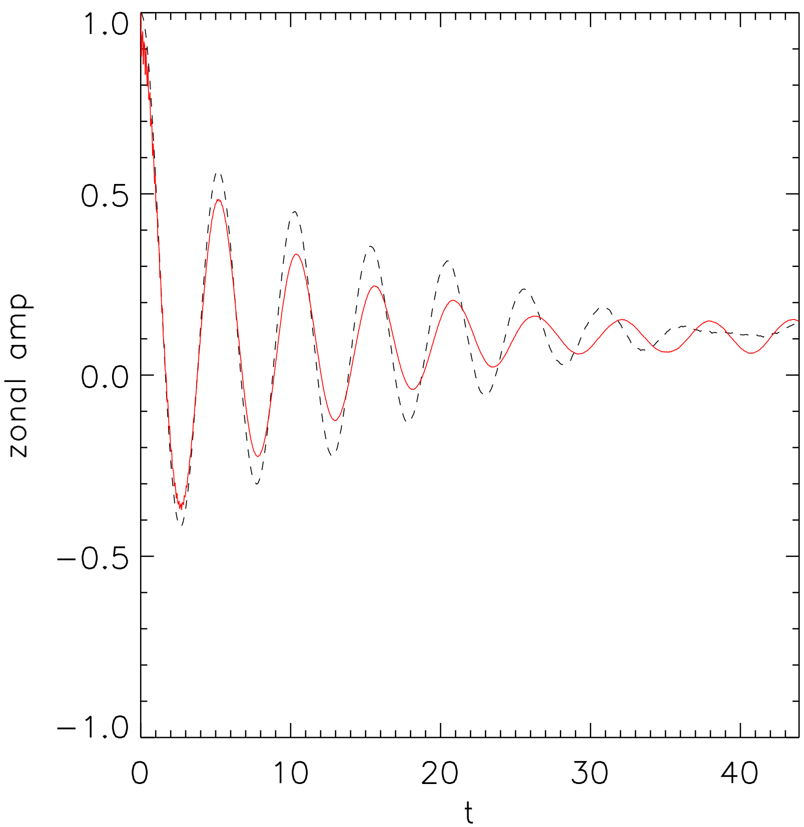


Finite-β Important in Determining Transport Level

- DIII-D Cyclone Base Case parameters: $R/L_{\mathrm{Ti}} = 6.9$, $R/L_{\mathrm{Te}} = 0$, $R/L_n = 2.2$, $q_0 = 1.4$, $s = 0.78$, $r_0/R = 0.18$, $m_i = 1$, $m_e = 1/1837$.
- $l_x = 104\rho_i$, $l_y = 128\rho_i$, resolution $n_x \times n_y = 128 \times 128$. With collisions. $\triangle t\omega_{\mathrm{ci}} = 10,\ 4,\ 2.5$. Particle number 8 388 608 per species.
- Low-β results are converged with respect to particle number and box size. $\beta = 0.8\%$ case not tested.
- With kinetic electrons $\gamma L_n/v_{\mathrm{Ti}} = 0.21$, with adiabatic electrons $\gamma L_n/v_{\mathrm{Ti}} = 0.12$. The increase is due to trapped electrons.
- With kinetic electrons $\chi_i/\rho_i v_{\mathrm{Ti}} = 0.01$, significantly increased from that with adiabatic electrons, $\chi_i/\rho_i v_{\mathrm{Ti}} = 0.0065$.
- As a result of the nonadiabatic effect of the electrons, a finite particle number flux is observed $D_i/\rho_i v_{\mathrm{Ti}} \approx 0.016$.
- **At $\beta = 0.8\%$ χ_i is significantly reduced from the adiabatic level. The heat flux could be further reduced as β increases to just below the KBM threshold.**

Evolution of Zonal Flow and the Residual Level not Significantly Changed by Kinetic Electrons





Summary and Future Work

- Linear electromagnetic simulation of microinstabilities with kinetic electrons using the split-weight scheme is extended from the regime of $\beta_e \frac{m_i}{m_e} \leq 1$ to $\beta_e \frac{m_i}{m_e} \geq 20$, for instabilities on the ion Larmor radius scale.
- Benchmarking with **gks** and GYRO shows good agreement.
- The flux-tube based code includes both passing and trapped drift-kinetic electrons, gyrokinetic ions, electron-ion collisions.
- **Simulation of the DIII-D base case shows that finite-β effect important in determining the transport level**. χ_i changes with β from above the adiabatic level to below.
- The evolution of the sonal flow is not significantly changed by kinetic electrons.

Future Work

- β-scan of transport level.
- Better Ampere solver?
- Extending the algorithm to general geometry and using quasi-ballooning coordinates are underway using the Summit Framework.